Lecture 2.2: Basic Principles of Option Pricing

Some Important Concepts in Financial and Derivative Markets

- Risk Preference
  - Risk aversion vs. risk neutrality
  - Risk premium – an additional return a risk-averse investor expect to earn on average to take a risk.
- Short Selling
  - Short selling on a stock is selling a stock borrowed from someone else (e.g., a broker).
  - Short selling is done in the anticipation of the price falling, at which time the short seller would then buy back the stock at a lower price, capturing a profit and repaying the shares to the broker.

Important Concepts

- Some important concepts in financial and derivative markets
- Concept of intrinsic value and time value
- Concept of time value decay
- Effect of volatility on an option price
- Put-call parity

Arbitrage and the Law of One Price

- **Law of one price**: same good must be priced at the same price
- **Arbitrage** defined: A type of profit-seeking transaction where the same good trades at two prices → buy one at low price and sell the other with high price.
- Example: See Figure 1.2 -> The concept of states of the world
- The Law of One Price requires that equivalent combinations of assets, meaning those that offer the same outcomes, must sell for a single price or else there would be an opportunity for profitable arbitrage that would quickly eliminate the price differential.
Basic Notation and Terminology

- **Symbols**
  - \( S_0 \) = stock price today, where time 0 = today
  - \( X \) = exercise price
  - \( T \) = time to expiration in years = (days until expiration)/365
  - \( r \) = risk free rate
  - \( S_T \) = stock price at expiration
  - \( C(S_0, T, X) \) = price of a call option in which the stock price is \( S_0 \), the time to expiration is \( T \), and the exercise is \( X \)
  - \( P(S_0, T, X) \) = price of a put option in which the stock price is \( S_0 \), the time to expiration is \( T \), and the exercise is \( X \)

**Concept of intrinsic value:**

- **Intrinsic value (IV)** is the value the call holder receives from exercising the option. So IV is positive for in-the-money calls and zero for at- and out-of-the-money calls
  - \( S_0 = $125, X = $120 \), then IV = 5
  - \( S_0 = $120, X = $120 \), then IV = 0
  - \( S_0 = $118, X = $120 \), then IV = 0

**TABLE 3.1** DCRB Option Data, May 14

<table>
<thead>
<tr>
<th>Exercise Price</th>
<th>Calls</th>
<th>Puts</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>May</td>
<td>June</td>
</tr>
<tr>
<td>120</td>
<td>8.75</td>
<td>15.40</td>
</tr>
<tr>
<td>125</td>
<td>5.75</td>
<td>13.50</td>
</tr>
<tr>
<td>130</td>
<td>3.60</td>
<td>11.35</td>
</tr>
</tbody>
</table>

Current stock price: 125.94
Expiration: May 21, June 18, July 16
Principles of Call Option Pricing

- Concept of *time value*
  - The price of an American call normally exceeds its intrinsic value. The difference between the option price and the intrinsic value is called the *time value* or *speculative value*.
  - The time value/speculative value reflects what traders are willing to pay for the uncertainty of the underlying stock.
- See Table 3.2 for intrinsic and time values of DCRB calls
- The time value is low when the call is either deep in- or deep-out-of-the-money. Time value is high when at-the-money....
  - The uncertainty (about the call expiring in- or out-of-the-money) is greater when the stock price is near the exercise price.

<table>
<thead>
<tr>
<th>Exercise Price</th>
<th>Calls</th>
<th>Puts</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>May</td>
<td>June</td>
</tr>
<tr>
<td>120</td>
<td>8.75</td>
<td>15.40</td>
</tr>
<tr>
<td>125</td>
<td>5.75</td>
<td>13.50</td>
</tr>
<tr>
<td>130</td>
<td>3.60</td>
<td>11.35</td>
</tr>
</tbody>
</table>

Current stock price: 125.94
Expirations: May 21, June 18, July 16

<table>
<thead>
<tr>
<th>Exercise Price</th>
<th>Intrinsic Value ( \text{May} )</th>
<th>Intrinsic Value ( \text{June} )</th>
<th>Intrinsic Value ( \text{July} )</th>
<th>Time Value ( \text{May} )</th>
<th>Time Value ( \text{June} )</th>
<th>Time Value ( \text{July} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>120</td>
<td>5.94</td>
<td>2.81</td>
<td>9.46</td>
<td>14.96</td>
<td></td>
<td></td>
</tr>
<tr>
<td>125</td>
<td>0.94</td>
<td>4.81</td>
<td>12.56</td>
<td>17.66</td>
<td></td>
<td></td>
</tr>
<tr>
<td>130</td>
<td>0.00</td>
<td>3.60</td>
<td>11.35</td>
<td>16.40</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Principles of Call Option Pricing (continued)

- Concept of *time value decay*
  - As expiration approaches (i.e., short time remaining for an option), the call price loses its time value → “time value decay”.
  - At expiration, the call price curve collapses onto the intrinsic value → time value goes to zero at expiration.

- Effect of Stock Volatility
  - The higher the volatility of the underlying stocks, the higher the price of a call
  - Intuition....
    - If the stock price increases, the gains on the call increase.
    - If the stock price decreases, it does not matter since the potential loss on the call is limited.
Principles of Put Option Pricing

- **Concept of intrinsic value:**
  - **Intrinsic value (IV)** is the value the put holder receives from exercising the option. So IV is positive for in-the-money puts and zero for at- and out-of-the-money puts.
  - \(S_0 = \$125, X = \$120\), then \(IV = 0\)
  - \(S_0 = \$120, X = \$120\), then \(IV = 0\)
  - \(S_0 = \$118, X = \$120\), then \(IV = 2\)

- **Concept of time value**
  - The price of an American put normally exceeds its intrinsic value. The difference between the option price and the intrinsic value is called the **time value** or speculative value.
  - The time value/speculative value reflects what traders are willing to pay for the uncertainty of the underlying stock.
  - See Table 3.7 for intrinsic and time values of DCRB puts.
  - The time value is largest when the stock price is near the exercise price.

**TABLE 3.1** DCRB Option Data, May 14

<table>
<thead>
<tr>
<th>Exercise Price</th>
<th>Calls May</th>
<th>Calls June</th>
<th>Calls July</th>
<th>Puts May</th>
<th>Puts June</th>
<th>Puts July</th>
</tr>
</thead>
<tbody>
<tr>
<td>120</td>
<td>8.75</td>
<td>15.40</td>
<td>20.90</td>
<td>2.75</td>
<td>9.25</td>
<td>13.65</td>
</tr>
<tr>
<td>125</td>
<td>5.75</td>
<td>13.50</td>
<td>18.60</td>
<td>4.60</td>
<td>11.50</td>
<td>16.60</td>
</tr>
<tr>
<td>130</td>
<td>3.60</td>
<td>11.35</td>
<td>16.40</td>
<td>7.35</td>
<td>14.25</td>
<td>19.65</td>
</tr>
</tbody>
</table>

Current stock price: \(125.94\)
Expirations: May 21, June 18, July 16

**TABLE 3.7** Intrinsic Values and Time Values of DCRB Puts

<table>
<thead>
<tr>
<th>Exercise Price</th>
<th>Intrinsic Value</th>
<th>Time Value May</th>
<th>Time Value June</th>
<th>Time Value July</th>
</tr>
</thead>
<tbody>
<tr>
<td>120</td>
<td>0.00</td>
<td>2.75</td>
<td>9.25</td>
<td>13.65</td>
</tr>
<tr>
<td>125</td>
<td>0.00</td>
<td>4.60</td>
<td>11.50</td>
<td>16.60</td>
</tr>
<tr>
<td>130</td>
<td>4.06</td>
<td>3.29</td>
<td>10.19</td>
<td>15.59</td>
</tr>
</tbody>
</table>

Principles of Put Option Pricing (continued)

- **Concept of time value decay**
  - As expiration approaches (i.e., short time remaining for an option), the put price loses its time value → **“time value decay”**.
  - At expiration, the put price curve collapses onto the intrinsic value → time value goes to zero at expiration.
Principles of Put Option Pricing (continued)

- The Effect of Stock Volatility
  - The effect of volatility on a put’s price is the same as that for a call. Higher volatility increases the possible gains for a put holder.
    - If the stock price decreases, the gains on the put increase.
    - If the stock price increases, it does not matter since the potential loss on the put is limited.
  - The higher the volatility of the underlying stocks, the higher the price of a put.

Put-Call Parity: European Options

- The prices of European puts and calls on the same stock with identical exercise prices and expiration dates have a special relationship.
- **Portfolio A:** (1) Buying a put option with the same X as the call + (2) A share.

\[
\text{Cost of establishing the portfolio A: } \quad P + S_0 \quad \text{where } P = \text{price of a put to sell one share, } \quad S_0 = \text{current share price}
\]

- At maturity, if \( S_T > X \), the put option is expired worthless, and the portfolio is worth \( S_T \).
- At maturity, if \( S_T < X \), the put option is exercised at option maturity, and the portfolio becomes worth \( X \).

Put-Call Parity: European Options

- **Portfolio B:** (1) Buying a call option + (2) buying risk-free zero-coupon T-bills with face value equal to the exercise price of the call (X)

\[
\text{Cost of establishing the portfolio B: } \quad C + \frac{X}{(1+r)^T} \quad \text{where } C = \text{price of a call to buy one share}
\]

- the T-bills will worth \( X \) at the maturity.
- At maturity, if \( S_T > X \), the call option is exercised and **portfolio A is worth** \( S_T \).
- At maturity, if \( S_T < X \), the call option expires worthless and the **portfolio is worth** \( X \).
Put-Call Parity: European Options

- Both portfolios have the same outcomes at the options’ expiration. Thus, it must be true that
  \[ S_0 + P_e(S_0, T, X) = C_e(S_0, T, X) + X(1+r)^{-T} \]
  This is called put-call parity.
  - A share of stock plus a put is equivalent to a call plus risk-free bonds.
  - Owning a call is equivalent to owning a put, owning the stock, and selling short the bonds (i.e., borrowing).
  - Owning a put is equivalent to owning a call, selling short the stock, and buying the bonds (i.e., lending).
  - \[ P_e(S_0, T, X) = C_e(S_0, T, X) - S_0 + X(1+r)^{-T} \]

A put is equivalent to owning a call, selling short the stock, and buying the bonds (i.e., lending).

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>Action</th>
<th>Payoffs from Portfolio given stock price at expiration</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>( S_T \leq X )</td>
</tr>
<tr>
<td>( A )</td>
<td>( P_e(S_0, T, X) )</td>
<td>( X - S_T )</td>
</tr>
<tr>
<td>( B )</td>
<td>( C_e(S_0, T, X) )</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>( -S_0 )</td>
<td>( -S_T )</td>
</tr>
<tr>
<td></td>
<td>( X(1+r)^{-T} )</td>
<td>( X )</td>
</tr>
<tr>
<td></td>
<td>( X - S_T )</td>
<td>0</td>
</tr>
</tbody>
</table>

A call is equivalent to owning a put, owning the stock, and selling short the bonds (i.e., borrowing).

**Arbitrage**

- **Example:** Suppose that \( S_0 = 31 \), \( X = 30 \), and \( r = 10\% \) per annum, \( C = 3 \), and \( P = 2.25 \), \( T = 3/12 \). The stock pays no dividend.

\[
C + \frac{X}{(1+r)^T} = 3 + \frac{30}{(1 + 0.10)^{3/12}} = \$32.29
\]

\[
Portfolio A: \quad C + \frac{X}{(1+r)^T} = 3 + \frac{30}{(1 + 0.10)^{3/12}} = \$32.29
\]

\[
Portfolio B: \quad P + S_0 = 2.25 + 31 = \$33.25
\]

Arbitrage strategy: **\( B \) is overpriced relative to \( A \)**

- **Buy the securities in portfolio \( A \) ⇒ buy the call and T-bills**
- **Short the securities in portfolio \( B \) ⇒ short the put and the stock**

Today:
- this strategy will generate the profit of \((\$33.25) - (\$32.29) = \$0.95\)**
Arbitraging

Long Portfolio A: buy the call and T-bills
Short Portfolio B: short the put and the stock

<table>
<thead>
<tr>
<th>Position</th>
<th>Immediate Cash Flow</th>
<th>Cash flow in the next 3 months</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>( S_T \leq X )</td>
</tr>
<tr>
<td>Buy call</td>
<td>-3</td>
<td>0</td>
</tr>
<tr>
<td>Buy bond</td>
<td>-29.29</td>
<td>30</td>
</tr>
<tr>
<td>Sell put</td>
<td>2.25</td>
<td>-(30 - ( S_T ))</td>
</tr>
<tr>
<td>Sell stock</td>
<td>31</td>
<td>-( S_T )</td>
</tr>
<tr>
<td>TOTAL</td>
<td>0.95</td>
<td>0</td>
</tr>
</tbody>
</table>

In 3 months:
- **If** \( S_T < X \)
  - Put will be exercised. Thus, the put holder has an obligation to buy a stock at \( X \), T-bills will be worth \( X \)
  - The stock exercised (worth \( S_T \)) will be returned to the broker (to satisfy the prior short sale position).
- **If** \( S_T > X \)
  - Call will be exercised to buy a stock at \( X \), T-bill will be worth \( X \)
  - The stock exercised will be returned to the broker (to satisfy the prior short sale position).
- Buying and selling pressure resulted from the arbitrage will restore the parity condition.

Initially, \( C + \frac{X}{(1 + r)^T} < P + S_0 \) — Short

---

TABLE 3.12 Put-Call Parity for DCRB Calls

<table>
<thead>
<tr>
<th>Exercise Price</th>
<th>May</th>
<th>June</th>
<th>July</th>
</tr>
</thead>
<tbody>
<tr>
<td>120</td>
<td>128.69</td>
<td>135.19</td>
<td>139.59</td>
</tr>
<tr>
<td>125</td>
<td>130.54</td>
<td>137.64</td>
<td>142.54</td>
</tr>
<tr>
<td>130</td>
<td>133.29</td>
<td>140.19</td>
<td>145.59</td>
</tr>
</tbody>
</table>

\( A \) European Put-Call Parity

- Top row of call: \( S_T + P(S_T, T, X) \)
- Bottom row of call: \( C(S_T, T, X) + X (1 + r)^{-T} \)
Figure 3.11: The Linkage between Calls, Puts, and Underlying Asset and Risk-Free Bonds