Lecture 2.2: Basic Principles of Option Pricing

Important Concepts

- Some important concepts in financial and derivative markets
- Concept of intrinsic value and time value
- Concept of time value decay
- Effect of volatility on an option price
- Put-call parity

Some Important Concepts in Financial and Derivative Markets

- Risk Preference
  - Risk aversion vs. risk neutrality
  - Risk premium – an additional return a risk-averse investor expects to earn on average to take a risk.
- Short Selling
  - Short selling on a stock is selling a stock borrowed from someone else (e.g., a broker).
  - Short selling is done in the anticipation of the price falling, at which time the short seller would then buy back the stock at a lower price, capturing a profit and repaying the shares to the broker.

Arbitrage and the Law of One Price

- **Arbitrage** and the **Law of One Price**
  - Law of one price: *same good* must be priced at the *same price*
  - Arbitrage defined: *A type of profit-seeking transaction where the same good trades at two prices* \(\Rightarrow\) buy one at low price and sell the other with high price.
  - Example: See Figure 1.2 -> The concept of states of the world
  - The Law of One Price requires that equivalent combinations of assets, meaning those that offer the same outcomes, must sell for a single price or else there would be an opportunity for profitable arbitrage that would quickly eliminate the price differential.
Some Important Concepts in Financial and Derivative Markets

Basic Notation and Terminology

Symbols
- $S_0$ = stock price today, where time $0$ = today
- $X$ = exercise price
- $T$ = time to expiration in years = (days until expiration)/365
- $r$ = risk free rate
- $S_T$ = stock price at expiration
- $C(S_0, T, X)$ = price of a call option in which the stock price is $S_0$, the time to expiration is $T$, and the exercise is $X$
- $P(S_0, T, X)$ = price of a put option in which the stock price is $S_0$, the time to expiration is $T$, and the exercise is $X$

Principles of Call Option Pricing

Concept of intrinsic value:
- **Intrinsic value (IV)** is the value the call holder receives from exercising the option. So IV is positive for in-the-money calls and zero for at- and out-of-the-money calls
- $S_0 = $125, $X = $120, then IV = 5
- $S_0 = $120, $X = $120, then IV = 0
- $S_0 = $118, $X = $120, then IV = 0

**TABLE 3.1** DCRB Option Data, May 14

<table>
<thead>
<tr>
<th>Exercise Price</th>
<th>Calls</th>
<th>Puts</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>May</td>
<td>June</td>
</tr>
<tr>
<td>120</td>
<td>8.75</td>
<td>15.40</td>
</tr>
<tr>
<td>125</td>
<td>5.75</td>
<td>13.50</td>
</tr>
<tr>
<td>130</td>
<td>3.60</td>
<td>11.35</td>
</tr>
</tbody>
</table>

Current stock price: $125.94
Expirations: May 21, June 18, July 16
Principles of Call Option Pricing

- Concept of time value
  - The price of an American call normally exceeds its intrinsic value. The difference between the option price and the intrinsic value is called the time value or speculative value.
  - The time value/speculative value reflects what traders are willing to pay for the uncertainty of the underlying stock.
  - See Table 3.2 for intrinsic and time values of DCRB calls
  - The time value is low when the call is either deep in- or deep-out-of-the-money. Time value is high when at-the-money….
    - The uncertainty (about the call expiring in- or out-of-the-money) is greater when the stock price is near the exercise price.

---

Principles of Call Option Pricing (continued)

- Concept of time value decay
  - As expiration approaches (i.e., short time remaining for an option), the call price loses its time value → “time value decay”.
  - At expiration, the call price curve collapses onto the intrinsic value → time value goes to zero at expiration.

---

Principles of Call Option Pricing (continued)

- Effect of Stock Volatility
  - The higher the volatility of the underlying stocks, the higher the price of a call
  - Intuition….
    - If the stock price increases, the gains on the call increase.
    - If the stock price decreases, it does not matter since the potential loss on the call is limited.
Concept of intrinsic value:

- **Intrinsic value** (IV) is the value the put holder receives from exercising the option. So IV is positive for in-the-money puts and zero for at- and out-of-the-money puts.

- $S_0 = $125, X = $120$, then IV = 0
- $S_0 = $120, X = $120$, then IV = 0
- $S_0 = $118, X = $120$, then IV = 2

Concept of time value:

- The price of an American put normally exceeds its intrinsic value. The difference between the option price and the intrinsic value is called the **time value** or **speculative value**.

- The time value/speculative value reflects what traders are willing to pay for the uncertainty of the underlying stock.
- See Table 3.7 for intrinsic and time values of DCRB puts.
- The time value is largest when the stock price is near the exercise price.

Concept of **time value decay**:

- As expiration approaches (i.e., short time remaining for an option), the put price loses its time value → **“time value decay”**.
- At expiration, the put price curve collapses onto the intrinsic value → time value goes to zero at expiration.
Principles of Put Option Pricing (continued)

- The Effect of Stock Volatility
  - The effect of volatility on a put’s price is the same as that for a call. Higher volatility increases the possible gains for a put holder.
    - If the stock price decreases, the gains on the put increase.
    - If the stock price increases, it does not matter since the potential loss on the put is limited.
  - The higher the volatility of the underlying stocks, the higher the price of a put.

Put-Call Parity: European Options

- The prices of European puts and calls on the same stock with identical exercise prices and expiration dates have a special relationship.
- Portfolio A: (1) Buying a put option with the same X as the call + (2) A share.
  
  Cost of establishing the portfolio A:
  
  \[ P + S_0 \] , where \( P \) = price of a put to sell one share
  \( S_0 \) = current share price

  - At maturity, if \( S_T > X \), the put option is expired worthless, and the portfolio is worth \( S_T \).
  - At maturity, if \( S_T < X \), the put option is exercised at option maturity, and the portfolio becomes worth \( X \).

- Portfolio B: (1) Buying a call option + (2) buying risk-free zero-coupon T-bills with face value equal to the exercise price of the call (X)

  Cost of establishing the portfolio B:
  
  \[ C + \frac{X}{(1+r)^t} \], where \( C \) = price of a call to buy one share

  - the T-bills will worth \( X \) at the maturity.
  - At maturity, if \( S_T > X \), the call option is exercised and portfolio A is worth \( S_T \).
  - At maturity, if \( S_T < X \), the call option expires worthless and the portfolio is worth \( X \).

TABLE 3.11 Put-Call Parity

<table>
<thead>
<tr>
<th>Payoff From</th>
<th>Current Value</th>
<th>Payoffs from Portfolio Given Stock Price at Expiration</th>
</tr>
</thead>
<tbody>
<tr>
<td>A Stock</td>
<td>( S_T )</td>
<td>( S_T ) ( S_T ) ( X - S_T ) 0 ( X ) ( S_T )</td>
</tr>
<tr>
<td>Put</td>
<td>( P(S_T; X) )</td>
<td>( X - S_T ) ( 0 ) ( X ) ( S_T )</td>
</tr>
<tr>
<td>Bonds</td>
<td>( X(1+n)^{-1} )</td>
<td>( X ) ( X ) ( X ) ( S_T )</td>
</tr>
</tbody>
</table>
Put-Call Parity: European Options

- Both portfolios have the same outcomes at the options’ expiration. Thus, it must be true that
  \[ S_0 + P_e(S_0,T,X) = C_e(S_0,T,X) + X(1+r)^{-T} \]

  This is called put-call parity.

- A share of stock plus a put is equivalent to a call plus risk-free bonds.
- Owning a call is equivalent to owning a put, owning the stock, and selling short the bonds (i.e., borrowing).
- Owning a put is equivalent to owning a call, selling short the stock, and buying the bonds (i.e., lending).
- \[ P_e(S_0,T,X) = C_e(S_0,T,X) - S_0 + X(1+r)^{-T} \]

A call is equivalent to owning a put, owning the stock, and selling short the bonds (i.e., borrowing).

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>Action</th>
<th>Payoffs from Portfolio given stock price at expiration</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>( S_T \leq X ) | ( S_T &gt; X )</td>
</tr>
<tr>
<td>( A )</td>
<td>( C_e(S_0,T,X) )</td>
<td>0 | ( S_T - X )</td>
</tr>
<tr>
<td>( B )</td>
<td>( P_e(S_0,T,X) )</td>
<td>( X - S_T ) | 0</td>
</tr>
<tr>
<td></td>
<td>( -S_0 )</td>
<td>( S_T ) | ( S_T )</td>
</tr>
<tr>
<td></td>
<td>( -X(1+r)^{-T} )</td>
<td>( -X ) | ( -X )</td>
</tr>
<tr>
<td></td>
<td>( X - S_T )</td>
<td>0 | ( S_T - X )</td>
</tr>
</tbody>
</table>

A put is equivalent to owning a call, selling short the stock, and buying the bonds (i.e., lending).

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>Action</th>
<th>Payoffs from Portfolio given stock price at expiration</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>( S_T \leq X ) | ( S_T &gt; X )</td>
</tr>
<tr>
<td>( A )</td>
<td>( P_e(S_0,T,X) )</td>
<td>( X - S_T ) | 0</td>
</tr>
<tr>
<td>( B )</td>
<td>( C_e(S_0,T,X) )</td>
<td>0 | ( S_T - X )</td>
</tr>
<tr>
<td></td>
<td>( -S_0 )</td>
<td>( S_T ) | ( S_T )</td>
</tr>
<tr>
<td></td>
<td>( X(1+r)^{-T} )</td>
<td>( X ) | ( X )</td>
</tr>
<tr>
<td></td>
<td>( X - S_T )</td>
<td>0 | ( S_T - X )</td>
</tr>
</tbody>
</table>

Arbitraging

- Example: Suppose that \( S_0 = 31 \), \( X = 30 \), and \( r = 10\% \) per annum, \( C = 3 \), and \( P = 2.25 \), \( T = 3/12 \). The stock pays no dividend.

\[
\text{Portfolio } A: \quad C + \frac{X}{(1+r)^{1/12}} = 3 + \frac{30}{(1+0.10)^{1/12}} = 32.29
\]

\[
\text{Portfolio } B: \quad P + S_0 = 2.25 + 31 = 33.25
\]

Arbitrage strategy: **B is overpriced relative to A**

- Buy the securities in portfolio \( A \) ⇒ **buy the call and T-bills**
- Short the securities in portfolio \( B \) ⇒ **short the put and the stock**

Today:

- this strategy will generate the profit of \((33.25)-(32.29) = 0.95\)
### Arbitraging

**Long Portfolio A:** buy the call and T-bills  
**Short Portfolio B:** short the put and the stock

<table>
<thead>
<tr>
<th>Position</th>
<th>Immediate Cash Flow</th>
<th>Cash flow in the next 3 months</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$S_T \leq X$</td>
</tr>
<tr>
<td>Buy call</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Buy bond</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sell put</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sell stock</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>TOTAL</strong></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

In 3 months:
- **If $S_T < X$**
  - Put will be exercised. Thus, the put holder has an obligation to buy a stock at $X$, T-bills will be worth $X$  
  - The stock exercised (worth $S_T$) will be returned to the broker (to satisfy the prior short sale position).
- **If $S_T > X$**
  - Call will be exercised to buy a stock at $X$, T-bill will be worth $X$  
  - The stock exercised will be returned to the broker (to satisfy the prior short sale position).
- Buying and selling pressure resulted from the arbitrage will restore the parity condition.

In 3 months:

- **If $S_T < X$**
  - Put will be exercised. Thus, the put holder has an obligation to buy a stock at $X$, T-bills will be worth $X$  
  - The stock exercised (worth $S_T$) will be returned to the broker (to satisfy the prior short sale position).
- **If $S_T > X$**
  - Call will be exercised to buy a stock at $X$, T-bill will be worth $X$  
  - The stock exercised will be returned to the broker (to satisfy the prior short sale position).
- Buying and selling pressure resulted from the arbitrage will restore the parity condition.

**Long:** Initially, $C + \frac{X}{(1 + r_f)^T} < P + S_0$  
**Short:**

### Table 3.12: Put-Call Parity for DCRB Calls

<table>
<thead>
<tr>
<th>Exercise Price</th>
<th>May</th>
<th>June</th>
<th>July</th>
</tr>
</thead>
<tbody>
<tr>
<td>120</td>
<td>129.69</td>
<td>135.19</td>
<td>139.59</td>
</tr>
<tr>
<td>125</td>
<td>130.54</td>
<td>137.44</td>
<td>142.54</td>
</tr>
<tr>
<td>130</td>
<td>133.29</td>
<td>140.19</td>
<td>145.59</td>
</tr>
</tbody>
</table>

**A. European Put-Call Parity**

Top row of cell: $S_T + P(T, S_T, X)$  
Bottom row of cell: $C(T, S_T, X) + X(1 + r_f)^T$
FIGURE 3.11  The Linkage between Calls, Puts, and Underlying Asset and Risk-Free Bonds

Call | Put-Call Parity | Put

Underlying Asset | Risk-Free Bond